ECON 4925 Autumn 2011
Lecture 3. Hydro power

Førsund (2005), 1-3, Førsund (2007) Chapter 3

Lecturer:
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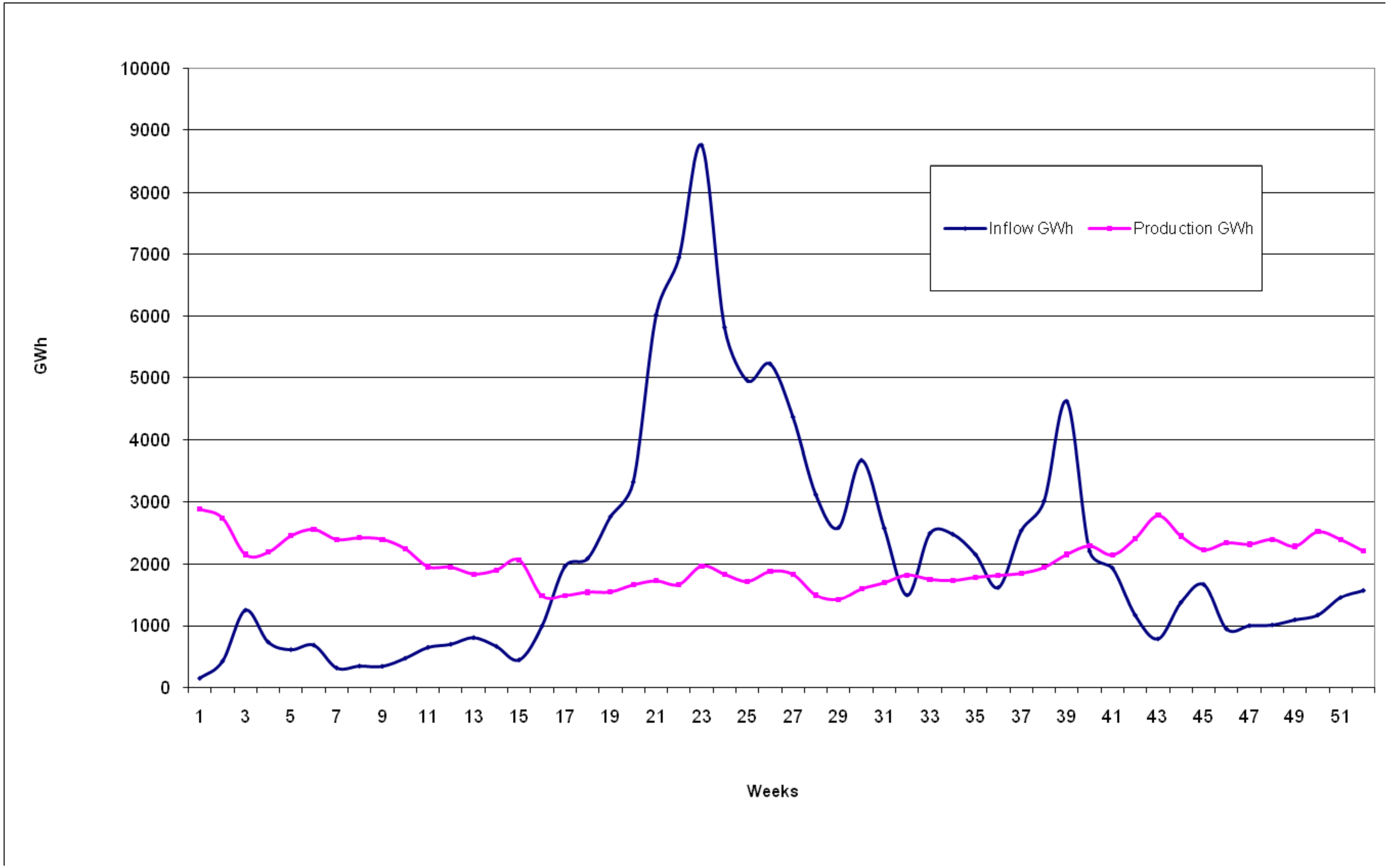
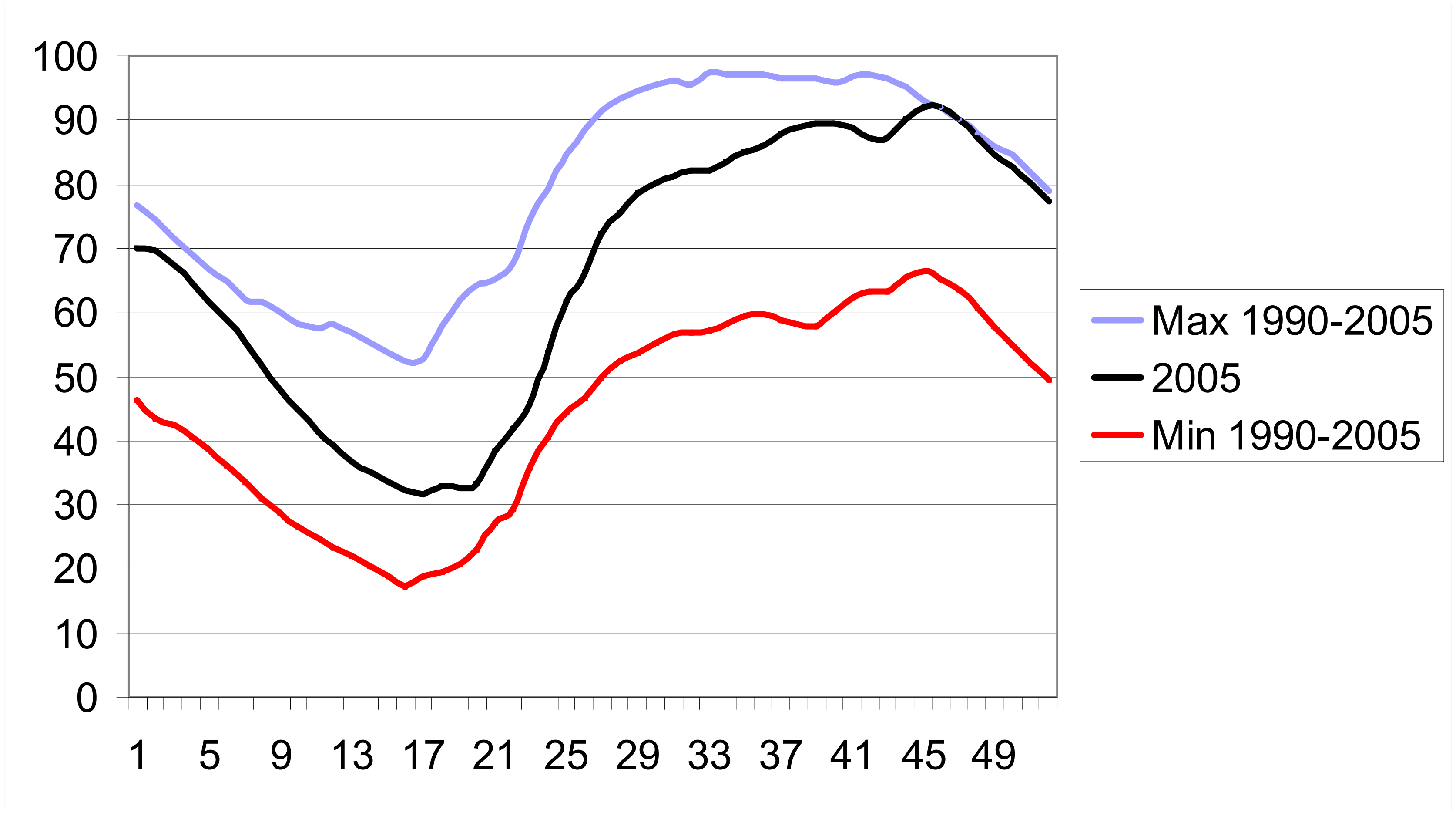
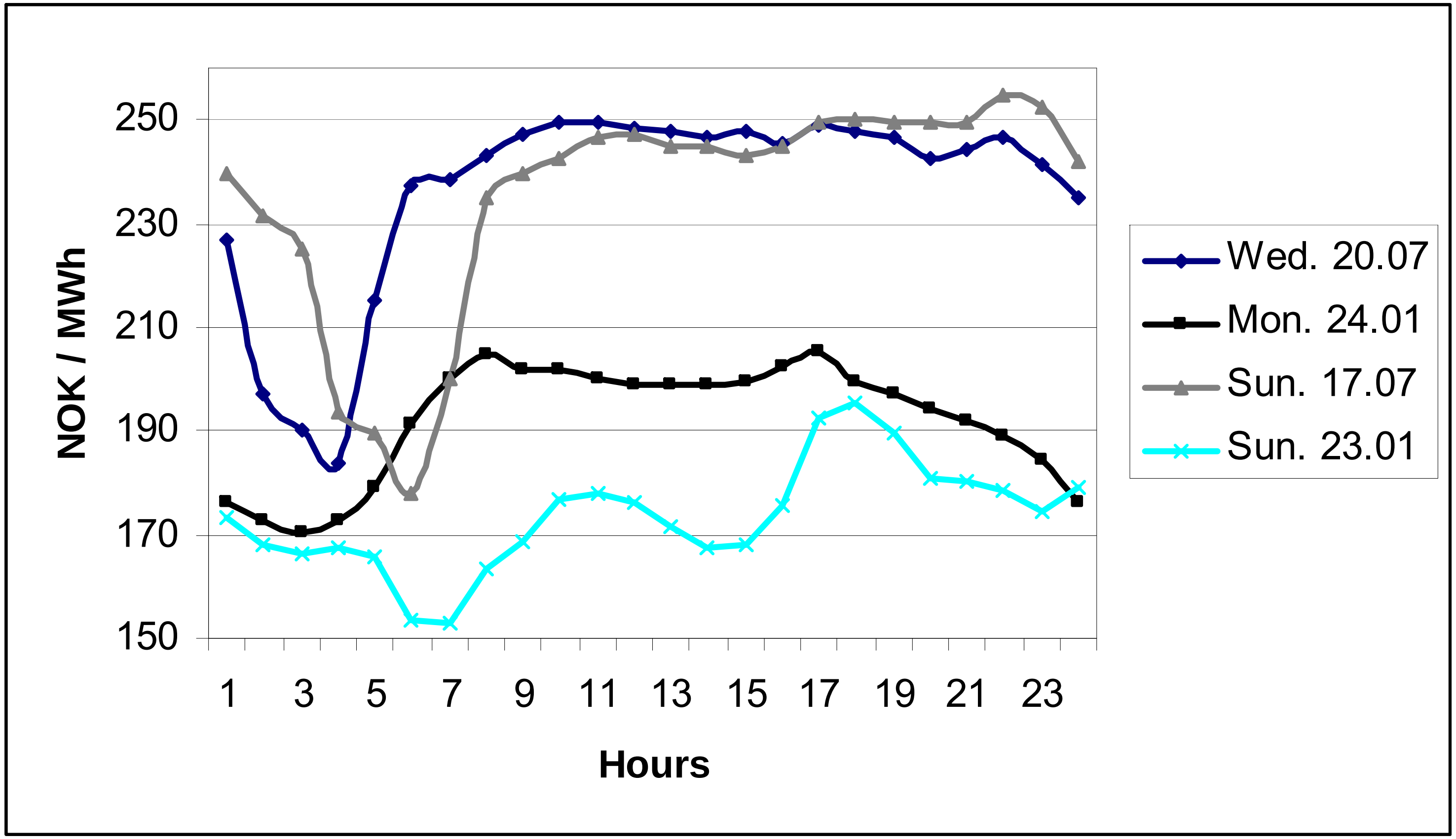
Weekly inflow and production of hydropower in Norway 2003



Degree of filling



Hourly price variation four days in 2005



Single plant with reservoir constraint

- The dynamics of water accumulation

$$R_t \le R_{t-1} + w_t - e_t^H, t = 1,..,T$$

- The reservoir constraint

$$R_t \le \bar{R}, t = 1,..,T$$

- The objective function: consumer + producer surplus

$$\sum_{t=1}^{T} \int_{z=0}^{e_t^H} p_t(z)dz$$

The social planning problem

$$Max \sum_{t=1}^{T} \int_{z=0}^{e_t^H} p_t(z)dz$$

$$s.t.$$

$$R_t \leq R_{t-1} + w_t - e_t^H$$

$$R_t \leq \overline{R}$$

$$R_t, w_t, e_t^H \geq 0 \ , \ \ t = 1,..,T$$

$$T, R_o \text{ given}, R_T \text{ free}$$

The Lagrangian function

$$L = \sum_{t=1}^{T} \int_{z=0}^{e_t^H} p_t(z)dz$$

$$-\sum_{t=1}^{T} \lambda_t (R_t - R_{t-1} - w_t + e_t^H)$$

$$-\sum_{t=1}^{T} \gamma_t (R_t - \overline{R})$$

Necessary first-order conditions

$$\frac{\partial L}{\partial e_t^H} = p_t(e_t^H) - \lambda_t \le 0 \ (= 0 \text{ for } e_t^H > 0)$$

$$\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \le 0 \ (= 0 \text{ for } R_t > 0)$$

$$\lambda_t \ge 0 (= 0 \text{ for } R_t < R_{t-1} + w_t - e_t^H)$$

$$\gamma_t \ge 0 (= 0 \text{ for } R_t < \bar{R}) \ , \qquad t = 1,..,T$$

Qualitative characterisations of the optimal solution

- Assumptions
  - Unique solution
  - Positive production of electricity in all periods
- Implication

$$p_t(e_t^H) = \lambda_t \ , t = 1,..,T$$

- λ:The shadow price of water, the water value

Backwards induction

- Start with terminal period

$$p_T(e_T^H) = \lambda_T \; (e_T^H > 0),$$

$$-\lambda_T - \gamma_T \leq 0 \; (= 0 \text{ if } R_T > 0)$$

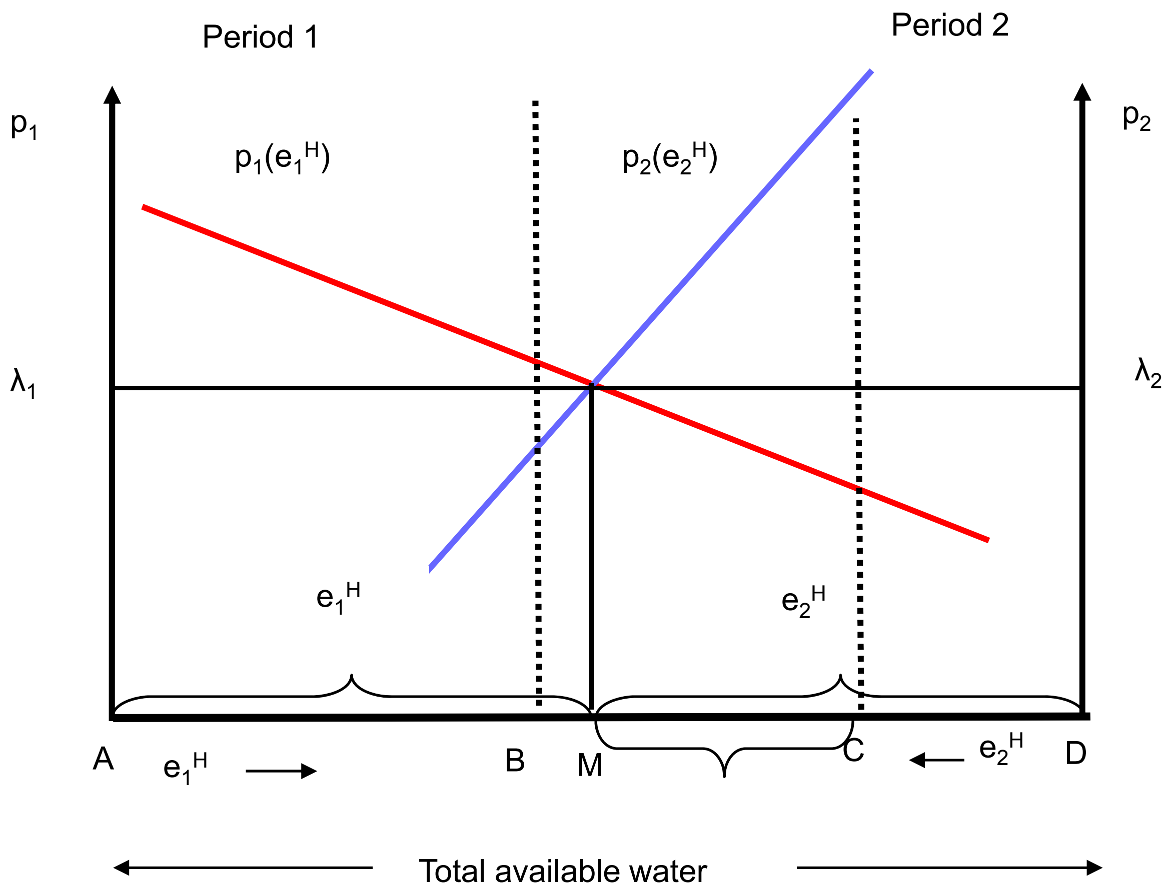
- Assumption

$$p_T(w_T + Max\, R_{T-1}) = p_T(w_T + \overline{R}) > 0$$

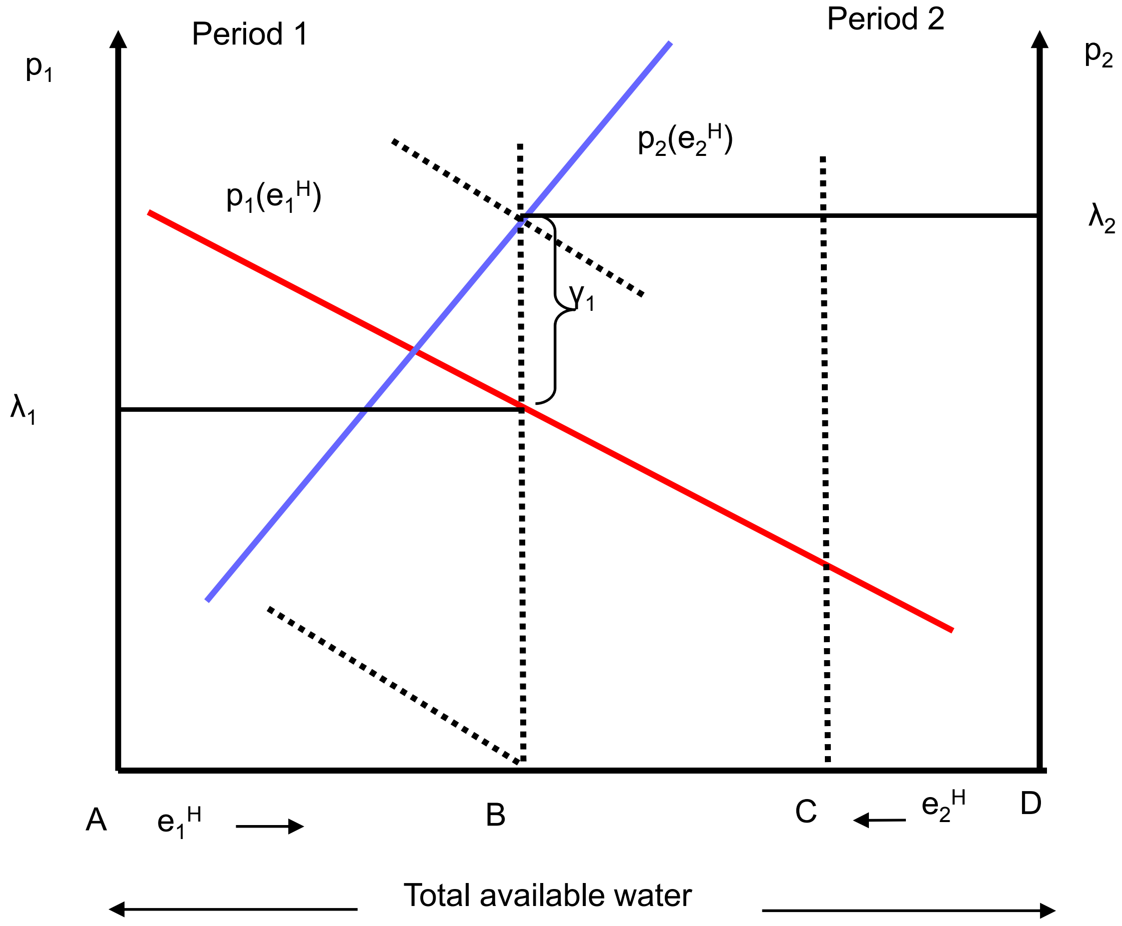
- Implication

$$R_T = 0,\; \gamma_T = 0,\; p_T(e_T^H) = \lambda_T > 0$$

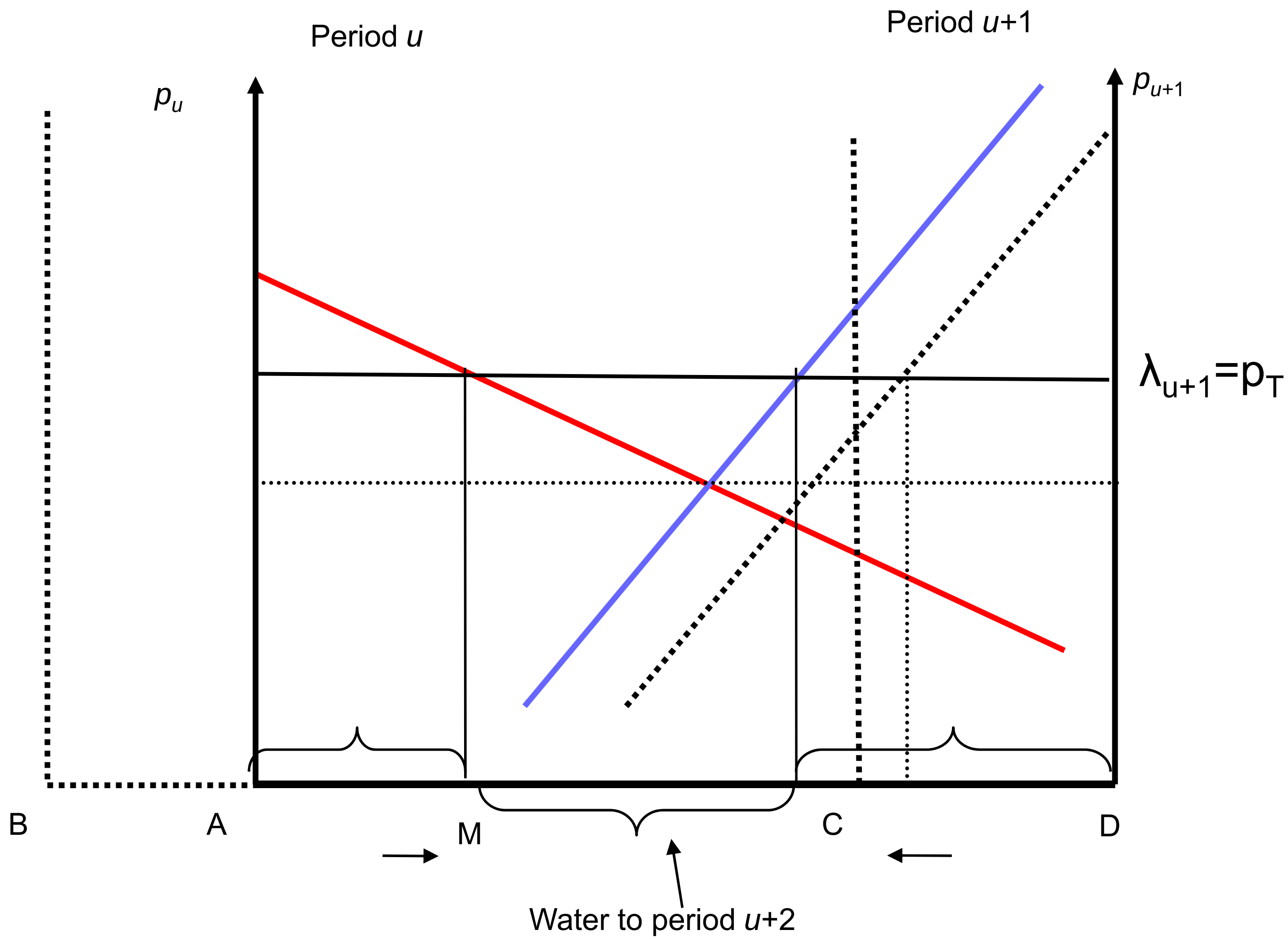
The bathtub diagram for two periods



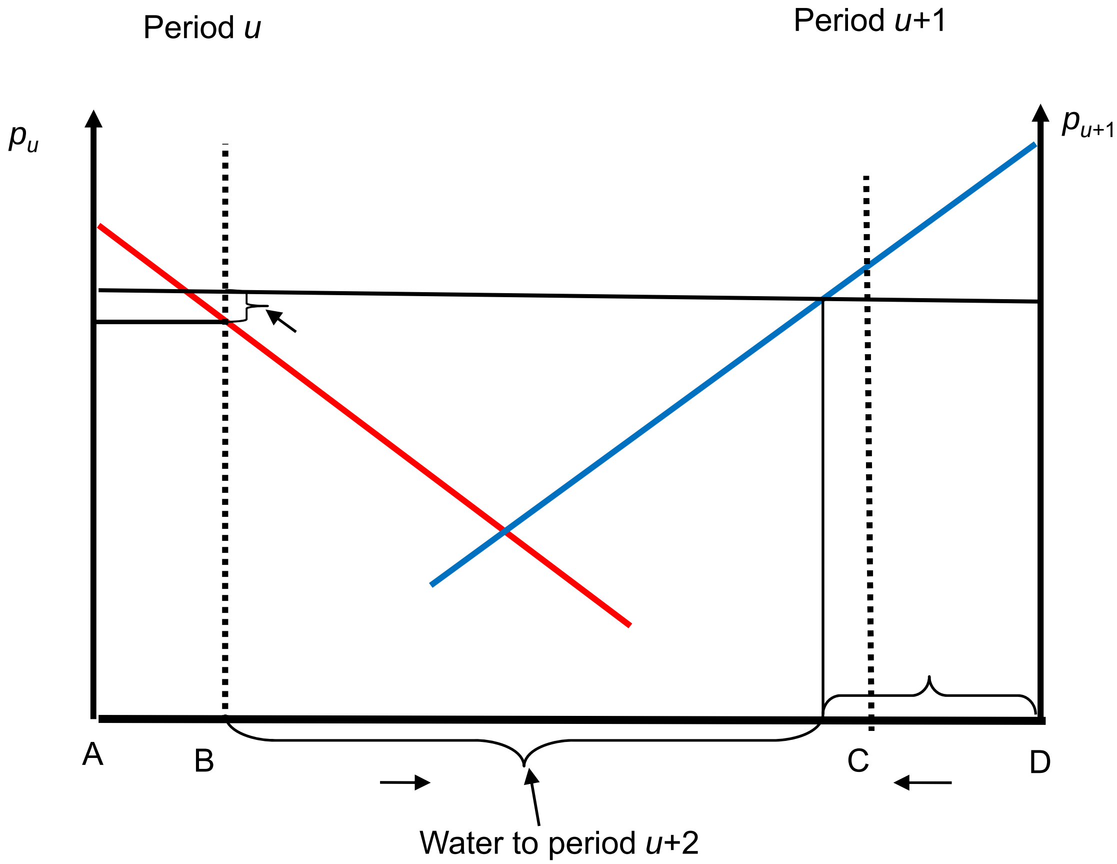
Threat of overflow



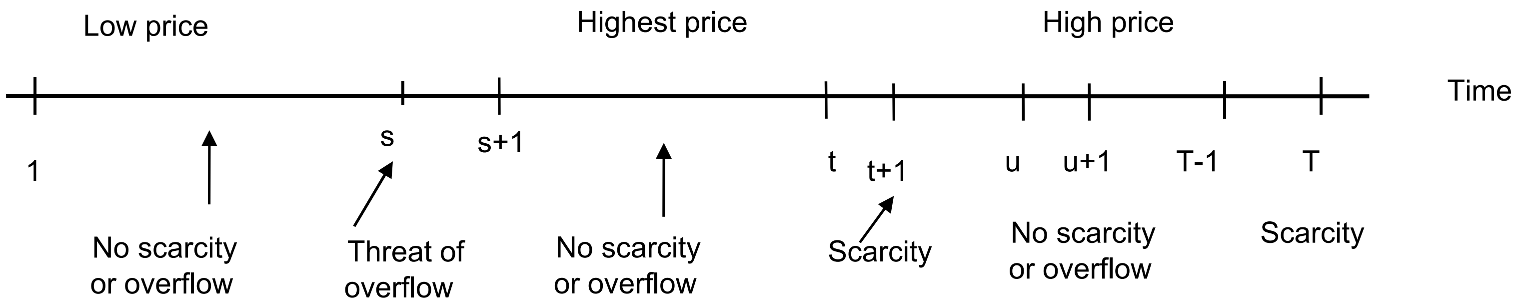
In between empty and full



Threat of overflow



Price-determining events



Must take: Run-of-river and wind power